The Paretian Optimum

Let us assume that the two consumers' utility functions are given by:

$$u_1 = u_1(q_{11}, q_{12}, q_{21}, q_{22})$$
$$u_2 = u_2(q_{11}, q_{12}, q_{21}, q_{22}) \qquad (21.19)$$

where $q_{11} + q_{21} = q_1^0$ and $q_{12} + q_{22} = q_2^0$.

Pareto optimality will be achieved if u_1 is maximum subject to a given level of $u_2 = u^0{}_2$.

In order to derive the conditions for this constrained maximisation, we have to form the Lagrange function:

$$L = u_1(q_{11}, q_{12}, q_{21}, q_{22}) + \lambda\,[u_2(q_{11}, q_{12}, q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0] \qquad (21.21)$$

The first order or necessary conditions for the constrained maximum u_1 are given by

$$\left.\begin{aligned}
\frac{\partial L}{\partial q_{11}} &= \frac{\partial u_1}{\partial q_{11}} - \frac{\partial u_1}{\partial q_{21}} + \lambda\left[\frac{\partial u_2}{\partial q_{11}} - \frac{\partial u_2}{\partial q_{21}}\right] = 0 \\
\frac{\partial L}{\partial q_{12}} &= \frac{\partial u_1}{\partial q_{12}} - \frac{\partial u_1}{\partial q_{22}} + \lambda\left[\frac{\partial u_2}{\partial q_{12}} - \frac{\partial u_2}{\partial q_{22}}\right] = 0 \\
\frac{\partial L}{\partial \lambda} &= u_2(q_{11}, q_{12}, q_1^0 - q_{11}, q_2^0 - q_{12}) - u_2^0 = 0
\end{aligned}\right] \qquad (21.22)$$

Now, from the first two equations of (21.22), we have

$$\frac{\dfrac{\partial u_1}{\partial q_{11}} - \dfrac{\partial u_1}{\partial q_{21}}}{\dfrac{\partial u_1}{\partial q_{12}} - \dfrac{\partial u_1}{\partial q_{22}}} = \frac{\dfrac{\partial u_2}{\partial q_{11}} - \dfrac{\partial u_2}{\partial q_{21}}}{\dfrac{\partial u_2}{\partial q_{12}} - \dfrac{\partial u_2}{\partial q_{22}}} \qquad (21.23)$$

Equation (21.23) is the necessary' condition for Pareto optimality in consumption when external effects are present. It generally differs from the Pareto optimality marginal condition as given by (21.18) or (21.16) or (21.11).

Perfect completion guarantees the attainment of (21.11) but not of (21.23). It is evident from (21.23) that if the external effects were absent, we would have $\partial u_1/\partial q_{21}$, $\partial u_1/\partial q_{22}$, $\partial u_1/\partial q_{11}$ and $\partial u_2/\partial q_{12}$, all equal to zero, and then (21.23) would have reduced to (21.11).

Since we have assumed here that the partial derivatives of the utility functions are functions of all variables, viz., q_{11}, q_{12}, q_{21} and q_{22}, the optimum position of each consumer depends upon the consumption level of the other.

For example, if we assume that the only external effect present in the two-consumer model is $\partial u_2/\partial q_{11}$ <; 0, then equation (21,23) becomes:

$$\frac{\frac{\partial u_1}{\partial q_{11}}}{\frac{\partial u_1}{\partial q_{12}}} = \frac{\frac{\partial u_2}{\partial q_{11}} - \frac{\partial u_2}{\partial q_{21}}}{-\frac{\partial u_2}{\partial q_{22}}}$$

$$= -\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} + \frac{\frac{\partial u_2}{\partial q_{21}}}{\frac{\partial u_2}{\partial q_{22}}}$$

i.e., MRS_{Q_1,Q_2} of consumer I $= -\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} + MRS_{Q_1,Q_2}$ of consumer II. (21.24)

Therefore, (21.24) gives us that, in this case of external effects, MRS_{Q_1,Q_2} of consumer II should be less than that of consumer I for an optimal distribution, since $\frac{\frac{\partial u_2}{\partial q_{11}}}{\frac{\partial u_2}{\partial q_{22}}} < 0$. It may be intuitively understood why this is so. If consumer I's consumption of Q_1 increases, then the utility level of consumer II declines. This implies that the marginal significance of Q_1 to consumer II is relatively large, which, again, implies that the

$MRS_{Q1,Q2}$ of consumer II should be smaller at the state of optimal distribution of the goods.

For, at this distribution as compared with the MRS-equating distribution, the quantity of Q_1 possessed by consumer II would be larger than that possessed by consumer I.

It can be shown diagrammatically with the help of Fig. 21.3 that condition (21.16) does not necessarily ensure Pareto optimality in the presence of external effects. Figures 21.3(a) and 21.3(b) give us the indifference map of consumers I and II, respectively.

Let us assume that initially, consumer I consumes the combination A and consumer II consumes the combination E.